XXIII. *Experiments on the Friction between Water and Air.* *By Dr.* VIKTOR VON LANG, *Professor of Physics in the University of Vienna. Communicated by* N. S. MASKELYNE, *M.A., F.R.S., Professor of Mineralogy in the University of Oxford.*

Received February 28,—Read April 6, 1876.

WE have only a few notices on the subject of the friction between air and water, and these few contradict each other. J. B. VENTURI* states that the air put into motion by a jet of water moves light bodies. G. MAGNUS† opposes this, as he finds that a flame brought near to a vertical jet of water indicates no motion of the surrounding air. The following experiments show, however, that great friction does really take place between water and air, so great as scarcely to differ from total adhesion.

But it is very much to be doubted whether VENTURI was acquainted with this pure friction between water and air, as he does not say any thing about the constitution of his jets. It must be remembered that VENTURI experimented long before SAVART, who was the first to distinguish the two parts of a vertical jet of water, the continuous part and the other broken up into drops. The above-mentioned phenomenon can be demonstrated only on the first part, though the discontinuous part of the jet moves the surrounding air too, and with much more energy. But in this case MAGNUS was quite right when he attributed the motion of the air to other causes.

Description of the Apparatus (Plate 52. figs. 1 & 2).

After what I have just said it is clear that I had to begin my experiments with producing a continuous vein of water. MAGNUS gives the necessary directions; but he lets the jet flow immediately from the reservoir, whilst I wished to use directly the conduit by which the city of Vienna has recently been supplied with water from the mountains. There is no doubt that in this way many inconveniences would be avoided, that make the method of MAGNUS and his predecessor SAVART tolerably troublesome. After a good many failures I succeeded at last, principally by inserting a caoutchouc tube between the water-tap and the delivering tube A.

The latter tube is of glass, 8 centims. long, outer radius 0·72 centim., inner radius 0·54 centim. It is not plane at the lower end, but irregularly indented.

The whole delivering apparatus must be set up in a very solid way; even the slightest shake is sufficient to break up the continuous jet into single drops; besides one must

* Recherches expérimentales sur le principe de communication latérale dans les fluides, appliqué à l'explication de différents phénomènes hydrauliques. Paris, 1797.

† "Über die Bewegung der Flüssigkeiten," POGGENDORFF's Annalen der Physik und Chemie, Band lxxx. (1850).

have the means of adjusting the delivering tube to a vertical position in order to avoid a displacement of the axis of the jet when the quantity of the water augments or diminishes. Both purposes may be attained in the following way:—

A wooden beam (B) is horizontally attached to the two walls forming the sides of the recess of a window. On it is fixed a brass plate (C) by means of two pulling and two pushing screws. The plate supports a vertical tube (D) of brass, 4 centims. long, and with an outer radius of 1·1 centim.; and this tube holds the delivering tube, which is cemented into it air-tight, and of course stands out at both ends.

By means of the above-mentioned four screws the delivering tube can be rendered vertical, remaining at the time totally independent of the vibrations of the floor.

The caoutchouc tube E that unites the delivering tube with the water-tap F below it is about one metre long and 0·3 centim. thick. The distance of the two ends of the caoutchouc tube is in vertical direction 30 centims., in horizontal direction 40 centims.; strings fastened to the ceiling hold up the tube.

By these arrangements it was possible to have continuous veins (W) of water of different lengths. The maximum lengths, however, could not be determined, the jet meeting, 130 centims. below the orifice, a board that was also fixed to the walls. If the quantity of water was so regulated by the tap F that the jet remained continuous till it reached the surface of the water in a vessel put under it, nothing was to be heard of the movement of the water, and the vein looked like a glass rod.

This regularity was of course disturbed in the most unpleasant way as often as the water-pipes were made use of in neighbouring places. A pressure-gauge inserted into the conduit indicated, however, when the normal state was restored. This normal pressure of the water at the place of observation is four atmospheres.

I have to describe now the special contrivances for the demonstration of the friction between the air and the continuous vein of water. There is first of all a crosspiece (G) made of glass tubes with an outer radius of 1·1 centim. One of the four arms of this crosspiece has a length of 36 centims., the other three a length of 7 centims. The long arm is to be united air-tight by a caoutchouc tube with the lower end of the brass tube D, both tubes having on this account the same outer radius. In this position of the crosspiece two arms of it stand horizontally. An open manometer is fastened to one of these arms by means of a caoutchouc connexion; the other arm leads to the "measuring tube" I, with which it is connected air-tight by a short caoutchouc tube. This tube I serves to measure the motion of the air caused by its friction on the jet of water in the "aspirating tube" K, the section of which does not very much exceed the section of the water vein. The aspirating tube is held up by the fourth arm of the crosspiece, with which it is united air-tight by a caoutchouc mouthpiece (L), such as is used for nursing children. The upper end of this tube was always brought to the height of the horizontal axis of the crosspiece, 35 centims. below the delivering orifice; the lower end was generally widened like a funnel to avoid disturbances by adhering drops. For the purpose of adjusting the aspirating tube so as to make its axis coincide with the axis of the water vein, I placed a heavy stand (M) with three movable arms on the above-mentioned board. Each of these arms ends in a ring with three screws, to hold a tube in the middle. The uppermost ring (N) serves only for securing the crosspiece, the longest arm of which is held up by it. As to the rest this arm does not differ from similar contrivances for chemical purposes. For the other two rings (O, O′), however, that have to hold the aspirating tube, one must have two horizontal motions perpendicular to each other and worked by screws. This is done in the following manner:—The sliding piece P, that belongs to ring O, bears a four-sided horizontal hole, into which fits a four-sided brass bar Q, the posterior end of which is worked into a screw. The nut R to this screw is held by a fork S fixed to the sliding piece, and the turning of this nut makes the bar move forward and backward. On one side of this bar is fixed the end of a strong spring T that bears in front the ring O. The spring presses against a screw U that goes horizontally through the four-sided bar, and that brings the ring to move nearly in a straight line from right to left.

About the measuring tube I have still to say that it forms a knee with two unequal arms. The shorter arm has also an outer radius of 1·1 centim., and is united, as I mentioned before, with one of the horizontal arms of the crosspiece by a short caoutchouc tube. The other arm of the knee is generally directed downward, and forms the proper measuring tube; its radius was taken of various sizes, exact measurements requiring a larger radius; it was then necessary to make the measuring tube of two pieces, that were united air-tight by cork and sealing-wax.

The motion of the air caused by the aspiration of the jet of water can now easily be rendered visible by means of soap-laminæ. If the open end of the measuring tube is immersed in a solution of soap, and if we remove this solution slowly, it leaves behind a lamina that immediately begins to follow the motion of the air in the aspirating tube. The whole measuring tube being wetted inside, such laminæ travel even through the horizontal part of the tube, till they burst at the mouth of the aspirating tube. One can also get a nearly infinite number of laminæ to ascend one after the other at the same time; and in this case the laminæ keep still better, as the velocity of the whole movement decreases with their number.

If we want to measure* the volume of the aspirated air, we shall employ a single lamina, and fix on the measuring tube a paper scale, by which we can determine the course of the lamina in one second. This quantity and the inner radius of the tube give the aspirated volume most easily.

There is still another way to study the motion of air caused by a jet of water by means of the described apparatus. In this case the measuring tube is replaced by one of metal. The open end of it is directed downward, and the flame of a turpentine-lamp is held in it. The intense smoke of such a flame reveals immediately vehement motion where all seems to be calm. The smoke ascends at the sides of the crosspiece, and descends near

* This method of measuring volumes of gases was already employed by Dr. T. Erner in his "Researches on the Diffusion of Gases through Liquid Laminæ," Sitzungsberichte der Wiener Akademie, Band lxx. (1874).

the jet of water. There is, of course, an intermediate zone without vertical motion; but in this zone a very violent rotation takes place around a horizontal axis perpendicular to the radius. This is shown by larger particles of soot that come by chance into this zone.

As to the motion of the air in the aspirating tube, the smoke is not dense enough to be seen in it when the radius of the tube is very small; but one sees that the smoke, in leaving the tube, continues to surround the jet for some time. This shows that the particles of air flow through the tube in parallel lines very regularly.

The last phenomenon does not take place any more when the radius of the aspirating tube becomes larger. In this case one can also see the smoke, and observe by it that the motion of the air is now very irregular, although no ascending movement can be detected.

Determination of the Form of the Water-jet.

The quantity of the aspirated air varies of course with the radius of the jet and with the velocity of the water on the surface of it. For finding this latter velocity there was nothing to be done but to suppose the velocity constant for the whole section of the jet. The velocity may then be calculated easily from the radius of the jet and from the volume of water discharged in one second. These two quantities depend on one another for each section of the jet. Having made corresponding measurements of them in different places of the jet, we may afterwards compute the radius only from the quantity of water discharged.

The measuring of the radius of the jet was effected by means of a kathetometer placed at a distance of 2 metres from the jet. The telescope bears an ocular micrometer, and its magnifying power is such that the distance of 1 centim. needs 6·80 turns of the micrometer-screw. I have to add that the quantity of water per second was found by weighing the quantity discharged in 5 seconds.

It would be sufficient for what follows to know the form of the jet from 35 centims. below the orifice. But it seems to me that for other problems it might perhaps be of interest to know the form of a jet of water beyond that limit also, and therefore I shall give my complete observations here.

In the following Table (p. 593) z denotes the vertical distance from the orifice, W the quantity of water discharged in one second, and r the radius of the jet at the place given by z. The distance z is measured in centimetres, W in grammes, and r in turns of the micrometer-screw. The numbers given for W and r are means from two to ten observations.

The column headed r' gives the values of r calculated by the following formulæ:—

$$z=35 \qquad r=1{\cdot}1860+0{\cdot}045202\ \mathrm{W}.$$
$$45 \qquad 1{\cdot}1503+0{\cdot}041941\ \mathrm{W}.$$
$$55 \qquad 1{\cdot}0566+0{\cdot}041354\ \mathrm{W}.$$
$$65 \qquad 1{\cdot}0606+0{\cdot}036555\ \mathrm{W}.$$
$$75 \qquad 0{\cdot}7064+0{\cdot}049137\ \mathrm{W}.$$

z.	W.	r.	r'.	r—r'.
5	2·2	1·29		
	4·0	1·56		
	8·0	2·24		
	12·0	2·72		
	15·8	2·98		
	18·5	3·24		
	23·1	3·67		
	29·5	4·07		
15	2·5	0·88		
	4·7	1·03		
	9·0	1·76		
	13·5	2·13		
	23·0	2·60		
	26·6	2·99		
	33·8	3·40		
25	5·4	1·20		
	9·4	1·52		
	13·0	1·80		
	19·0	2·13		
	24·0	2·45		
	29·7	2·75		
	33·5	2·95		
35	9·0	1·29		
	11·5	1·60		
	13·8	1·68		
	18·2	2·02	2·009	+0·011
	25·4	2·31	2·334	−0·024
	31·0	2·60	2·587	+0·013
45	8·1	1·25		
	11·2	1·50		
	13·8	1·68		
	18·0	1·85	1·905	−0·055
	24·2	2·25	2·165	+0·085
	27·4	2·30	2·299	−0·001
	34·8	2·58	2·610	−0·030
55	10·4	1·27		
	11·9	1·36		
	14·2	1·51		
	16·9	1·80	1·755	+0·045
	20·7	1·92	1·921	−0·001
	25·6	2·09	2·115	−0·025
	32·4	2·37	2·396	−0·026
65	12·9	1·36		
	14·7	1·56		
	18·3	1·73	1·729	+0·001
	23·8	1·93	1·931	−0·001
	28·6	2·11	2·106	+0·004
	36·2	2·38	2·384	−0·004
75	12·4	1·27		
	16·2	1·51	1·502	+0·008
	21·5	1·80	1·763	+0·037
	28·8	2·01	2·097	−0·087
	33·2	2·38	2·338	+0·044

The constants of these formulæ have been determined by the method of least squares from the corresponding values of r found by direct observation. The last formulæ give for W=20 and W=30 the following numbers for r:—

W.	z.	r.	r'.	$r-r'$.
20	35	2·090	2·085	+0·005
	45	1·989	1·957	+0·032
	55	1·884	1·865	+0·019
	65	1·791	1·795	−0·004
	75	1·689	1·741	−0·052
30	35	2·542	2·567	−0·025
	45	2·408	2·404	+0·004
	55	2·297	2·287	+0·010
	65	2·157	2·198	−0·041
	75	2·181	2·129	+0·052

The calculated values of r denoted by r' were found by the formulæ

$$\text{W}=20 \qquad r=0{\cdot}9976+6{\cdot}4342\,\frac{1}{\sqrt{z}},$$

$$30 \qquad 1{\cdot}1819+8{\cdot}1973\,\frac{1}{\sqrt{z}},$$

the constants of which were also determined by the method of least squares from the foregoing values of r. The calculated numbers agree in this case too with the observed ones, as well as is necessary for the present purpose. From the last two formulæ the following equation,

$$r=0{\cdot}6290+0{\cdot}01843\ \text{W}+(2{\cdot}9080+0{\cdot}17631\ \text{W})\,\frac{1}{\sqrt{z}},$$

was finally deduced, which gives the radius of the jet for any height and quantity of water. Expressing the turns of the micrometer-screw by centimetres, we have, in accordance with the value given above for these turns,

$$r=0{\cdot}09246+0{\cdot}002709\ \text{W}+(0{\cdot}42748+0{\cdot}025918\ \text{W})\,\frac{1}{\sqrt{z}}.$$

Determination of the Volume of the Aspirated Air.

I have already explained the method used for measuring the volume of the aspirated air. However, the question arises whether, in following this method, the volume measured by the motion of a soap-lamina is really equal to the volume that is aspirated when the measuring tube is quite open. First of all the weight of the lamina might be of influence, acting in a sense contrary to the movement of the air. But the experiment can easily be arranged in such a way that the weight of the lamina favours the air's motion: one has only to put in the measuring tube with its open end upward; the soap-laminæ cannot now be produced directly, but a large cylinder must be put first of all into the solution of soap, and afterwards the lamina must be transferred from this cylinder to the measuring tube. This procedure can also be of use sometimes when the measuring tube is directed downward.

It was found by such experiments that the weight of the lamina is of no appreciable influence, as both positions of the measuring tube gave the same numbers for the volume of the aspirated air under similar circumstances. The downward position of the measuring tube was therefore constantly adopted in the following experiments, this position being the more convenient.

The radius of the measuring tube, however, is of great importance, and it will be seen by using tubes of different radii that the larger tubes give a greater volume. But this augmentation becomes continually smaller if we proceed with enlarging the measuring tube, and from a certain radius the volume increases no more. At this point we can state that the soap-lamina is without influence on the quantity of aspirated air.

We can make similar observations also on the water-manometer mentioned before. It does not indicate any difference of pressure when during the aspiration the measuring tube remains open; but if we close it by a soap-lamina, the manometer shows a diminution of pressure that becomes the more insensible as the radius of the tube increases.

The radius at which the influence of the section of the measuring tube vanishes depends certainly on the quantity of the aspirated air. A radius of 2·4 centimetres seems to be quite sufficient for my experiments.

The motion of a soap-lamina is of course much slower in larger tubes than in small ones, and can therefore be measured with greater accuracy. The employment of larger measuring tubes is more advantageous on this account also.

The following Table gives the radii of all the measuring tubes that were made use of:—

Meas. tube.	Radius.
I.	1·096 centim.
II.	4·00 centims.
III.	2·42 ,,
IV.	1·57 centim.
V.	1·13 ,,
VI.	0·85 ,,

These tubes may also be employed by couples, placing the second measuring tube in lieu of the manometer. A few experiments which I made in this way will be found among the following measurements.

The first part of these measurements was executed in July 1875 with the aspirating tubes 1–4, the length of which was gradually diminished to find the influence of this dimension. The volume of the air was at the time measured by tube I., the radius of which has not the necessary value, as was found by later experiments. But this circumstance, which is of great influence in regard to the absolute values of the volume of air, may be neglected in the present research. The result of these experiments is that the quantity of the aspirated air approaches to a maximum with the length of the tube. The maximum itself could not be reached exactly, even with the smallest tubes; but this arises certainly from the conical form of the water vein; for within certain limits the volume of the air increases when the vein becomes smaller, the quantity of the water remaining constant. The following Table gives the numbers found by these experiments, the quantity of water discharged being equal to 20 gallons. A is the volume of the aspirated air, l the length of the aspirating tube, and R its radius, found by the weight of mercury filling up the tube. The latter two quantities are given in centimetres, and A in cubic centimetres.

Asp. tube.	R.	l.	A.
1	0·254	47·2	17·1
		38·2	16·6
		26·0	15·1
		16·0	14·0
2	0·288	47·1	22·3
		37·3	21·8
		26·5	19·1
3	0·383	49·8	28·1
		42·6	25·8
		36·5	22·6
4	0·435	50·5	25·2
		44·0	23·1
		31·7	17·5

The numbers given for A are not directly observed; but at first the mean was taken of all readings for A above and below W=20, and then the required quantity was interpolated between these two means by simple proportion.

For the tubes 1 and 2 the diminution of the volume A is very small, although their length decreases by 10 centimetres. The tubes 3 and 4, on the contrary, seem to be by far too short to give the maximum of A. We may therefore say that an aspirating tube 47 centimetres long, W being equal to 20 gallons, gives the maximum of air only when its radius is below 0·29 centimetre.

The second series of measurements were executed in October of the same year with the aspirating tubes 5–9, all of them 48 centimetres long, and with the measuring tubes II.–VI. The values of A found by these different tubes are given in the following Table. There exists no material difference between the numbers found by the tubes II. and III.; even the coupling of these two tubes did not produce larger volumes. For the following calculations the mean was therefore taken of the values II. and III., and the numbers so found are given in the last column.

Asp. tube.	R.	W.	II.	III.	IV.	V.	VI.	II.+III.	A.
5	0·238	26·0	13·8	14·0	14·1	13·4	12·6	14·0	14·0
6	0·244	25·5	14·7	14·8	14·7	13·8	13·5	......	14·8
7	0·261	20·0	20·5	19·5	18·7	17·8	17·1	......	20·0
		30·0	17·9	17·0	16·6	16·3	15·8	......	17·5
8	0·287	17·0	24·5	25·6	21·3	17·9	17·7	......	25·1
		22·0	24·0	23·8	22·1	19·6	18·4	......	23·9
		30·0	22·5	22·7	20·7	19·7	19·5	......	22·6
9	0·303	20·0	26·0	25·3	21·4	21·7	......	......	25·7
		23·8	25·5	25·2	22·5	21·8	......	26·1	25·6
		30·0	24·7	25·1	22·6	21·9	......	......	24·9

Approximate Theory of the Experiments.

The exact theory of these experiments is rendered still more difficult by the fact that the water-jet has a different radius at different places. But to obtain at least an approximate solution of the problem we will assume a cylindrical form of the jet. In accordance with our smoke-experiments, we assume further that all particles of air flow through the aspirating tube in straight lines parallel to the tube's axis, which we take for the axis of x. The velocity of the air-particles has therefore a finite value u only for this axis, it being zero for any direction perpendicular to this axis.

In consequence of the last statement the pressure must be constant in each section of the aspirating tube. Now we have seen by the manometer that the pressure at the beginning of the tube is the same as at the end, viz. the pressure of the atmosphere; this leads us to suppose that the pressure does not vary at all in the whole length of the tube.

If we consider, further, that the motion of the air in the aspirating tube is a steady one and independent of time, and that no exterior forces exist, the hydrodynamical equations are reduced in the present case to

$$\frac{d^2u}{dy^2}+\frac{d^2u}{dz^2}=0, \quad \ldots \ldots \ldots \quad (1)$$

the equation of continuity becoming

$$\frac{d(\mu u)}{dx}=0. \quad \ldots \ldots \ldots \quad (2)$$

In this last equation μ denotes the density, which must be constant when the pressure is constant. This equation is therefore satisfied by our suppositions. As regards the equation (1) its integral is

$$u=\mathrm{L}\log q+\mathrm{M}, \quad \ldots \ldots \ldots \quad (3)$$

q denoting the distance from the axis of x. The constants L and M have to be determined by the motion of the air next to the tube and next to the water-vein. If we assume that no slipping takes place, the air adhering to the glass as well as to the water, we have the corresponding values,

$$\left.\begin{array}{l} q=\mathrm{R},\ u=0, \\ q=r,\ \ u=\mathrm{U}, \end{array}\right\} \quad \ldots \ldots \quad (4)$$

U being the velocity of the water at the surface of the jet, given by

$$\mathrm{W}=\pi r^2 \mathrm{U}. \quad \ldots \ldots \quad (5)$$

By the equations (4) we get

$$u=\frac{\mathrm{W}}{\pi r^2}\frac{\log \mathrm{R}-\log q}{\log \mathrm{R}-\log r}; \quad \ldots \ldots \quad (6)$$

and in consequence for the volume A of air flowing through the tube per second,

$$\mathrm{A}=\int_r^{\mathrm{R}} u\, 2\pi q dq=\mathrm{W}\left[\frac{\mathrm{R}^2-r^2}{2r^2(\log \mathrm{R}-\log r)}-1\right]. \quad \ldots \ldots \quad (7)$$

In order to compare our observations with this formula, we have still to fix what numerical value we shall assume for the radius of the jet. The simplest way is of course to take for r the mean of its values at the top and at the bottom of the tube; and, indeed, we get in this way a tolerably good result, as is shown by the following comparison of the observed values of A and of the calculated ones designated by A′.

Asp. tube.	r.	A′.	A.	A − A′.
5	0·1582	14·20	14·0	−0·20
6	0·1566	15·56	14·8	−0·76
7	0·1392	20·02	20·0	−0·02
	0·1708	17·22	17·5	+0·28
8	0·1300	24·57	25·1	+0·53
	0·1456	24·77	23·9	−0·87
	0·1708	22·71	22·6	−0·11
9	0·1392	28·06	25·7	−2·36
	0·1512	27·83	25·6	−2·23
	0·1708	26·19	24·9	−1·29

It is not before we come to tube 9 that the differences between the observed and the calculated numbers are greater than the possible errors of observation. But the radius of this tube is already beyond the limit within which our suppositions as to the motion of air hold good.

A still better accord is got by supposing that there is a slipping between the air and the jet of water, and also between the air and the aspirating tube. We get, then, in lieu of the conditions (4) the following ones,

$$\left.\begin{array}{ll} q=\mathrm{R}, & u=-\psi\frac{du}{dq}, \\ q=r, \ \mathrm{U} & u=-\xi\frac{du}{dq}, \end{array}\right\} \quad \ldots \ldots \quad (8)$$

where ψ is the coefficient of slipping for the water, and ξ this coefficient for the glass tube. These conditions give for u and L,

$$u=\frac{W}{\pi r^2}\left.\frac{\log R-\log q+\frac{\psi}{R}}{\log R-\log r+\frac{\psi}{R}+\frac{\xi}{r}}\right\}, \quad \ldots \quad (9)$$

$$L=W\frac{(R^2-r^2)\left(\frac{R^2}{r}+\psi\right)-Rr^2(\log R-\log r)}{Rr^2(\log R-\log r)+Rr\xi+r^2\psi} \quad \ldots \quad (10)$$

The last formula applied to the former observations gives ten linear equations for ξ and ψ, from which one finds, by the method of least squares, $\xi=\psi=0{\cdot}029$. With these numbers we now get the following differences between the observed values of A and the calculated ones:—

	+0·23	−0·26	+0·87	+0·85	+1·69
+0·20	+0·71	−1·86	−1·18	−0·40	

One sees that the accord is of the same degree as before, although the sum of the squares of the errors is now a little smaller, 10·01 against 17·46 as before. The equality of ξ and ψ could easily be explained, as the tube was always wet inside; but the values of ξ and ψ cannot be relied on at all, because they become even negative when the first seven observations only are used for their determination. The real values of ξ and ψ can only be found by taking into account the conical form of the jet.

I have also calculated the first series of observations by aid of the equation (7). The calculated values are of course all too great, as the measuring tube with which the observations were executed was too small; but still one can see that the calculation goes parallel to the observation as long as the limit is not passed within which our theory holds good.

Asp. tube.	*l*.	*r*.	A.	A′.	A−A′.
1	47·2	0·1393	17·1	18·5	− 1·4
	38·2	0·1408	16·6	17·8	− 1·2
	26·9	0·1435	15·1	16·9	− 1·8
	16·0	0·1463	14·0	16·1	− 2·1
2	47·1	0·1393	22·3	25·1	− 2·8
	37·3	0·1410	21·8	24·3	− 2·4
	26·5	0·1434	19·1	23·4	− 4·3
3	49·8	0·1389	28·1	45·1	−18·0
	42·6	0·1400	25·8	44·4	−18·6
	36·5	0·1412	22·6	43·9	−21·3
4	50·5	0·1391	25·2	57·2	−32·0
	44·0	0·1398	23·1	56·2	−33·1
	31·7	0·1422	17·5	54·8	−37·3

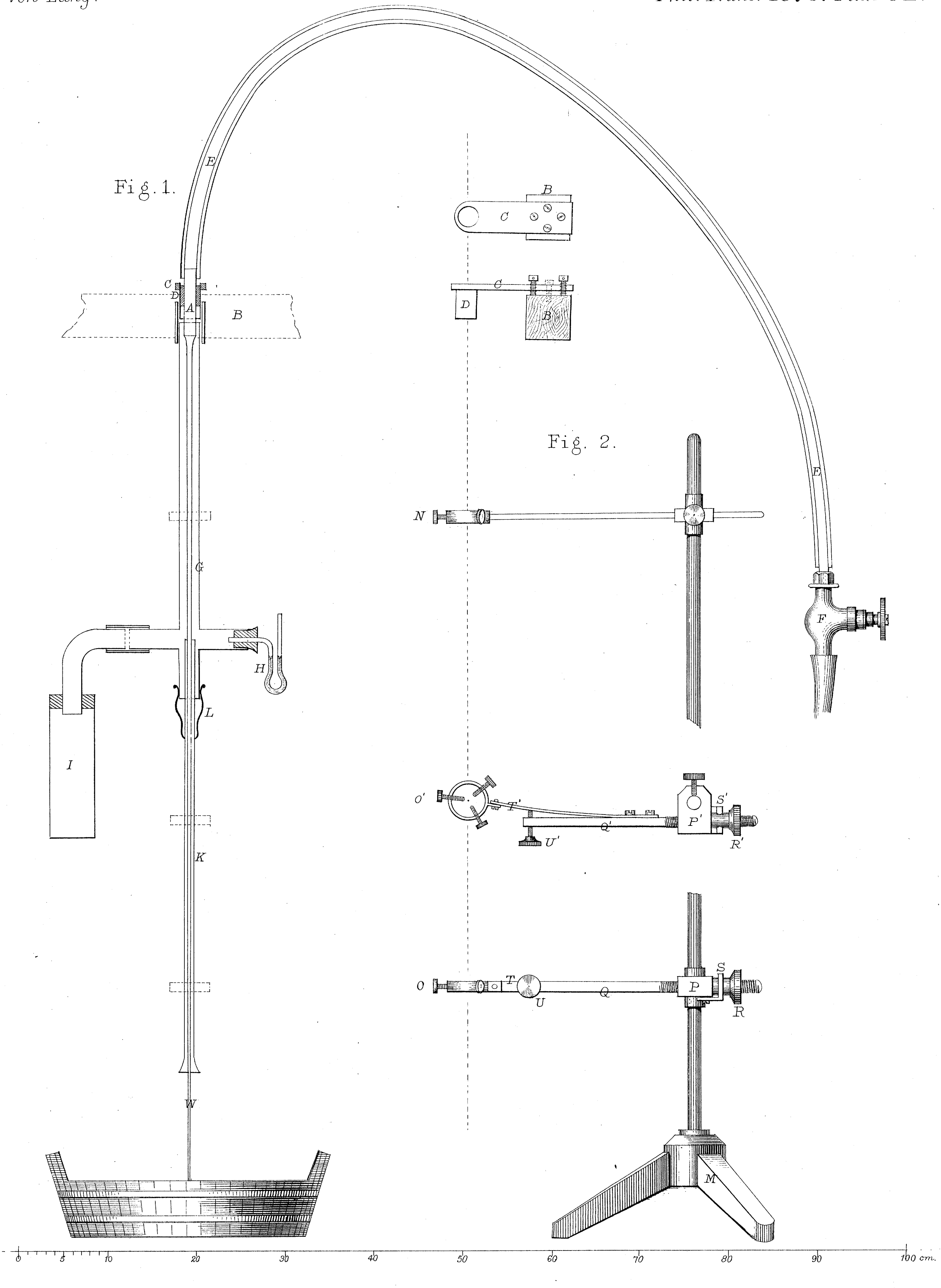
Appendix.

We have seen that even when we put $\xi = \psi = 0$, the differences between calculation and observation are not greater than the errors of observation. We therefore cannot expect essential differences when we employ aspirating tubes of other material. This was also confirmed by experiments made with two brass tubes. In this case it was found a little troublesome to make the jet pass along the axis of the tube, the tube being opaque. The radius of these tubes was determined by means of cylinders fitted carefully into the tubes; their length was again 48 centimetres. In the following Table the values of A are those found by observation, the values A′ those calculated by equation (7).

Asp. tube.	R.	W.	r.	A.	A′.	A−A′.
10	0·228	22·2	0·1461	14·7	13·66	+1·04
		24·9	0·1548	13·6	12·70	+0·90
11	0·270	19·8	0·1386	23·8	21·69	+2·11
		23·7	0·1509	23·2	21·14	+2·06

I have also made a few experiments with other gases. However, no great differences are to be expected in this case either, for the reason stated before, as differences could only arise from different coefficients of slipping. In these experiments the manometer-arm of the crosspiece was used for introducing the gas. In order to get the apparatus totally freed from air, the soap-lamina was alternately aspirated by the jet and driven back again to the mouth of the measuring tube by introduction of new gas. The results obtained are:—

Asp. tube.	W.	r.	A.	A′.	Name of gas.
6	24·3	0·1528	15·2	16·0	Coal-gas.
7	30·0	0·1708	17·0	17·2	,, ,,
8	24·2	0·1525	25·1	24·4	,, ,,
	26·0	0·1461	26·0	24·0	Carbonic acid.



W. West & Co. lith.